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SUSTAINABLE COVID-19 CONTROL MEASURES: A MATHEMATICAL MODELLING STUDY

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Abstract: Mathematical modelling techniques have become essential in predicting the consequences of viral disease outbreaks and in planning sustainable preventative measures to curb their transmission dynamics. To better understand the sustainable evolution of the control measures' impacts on COVID-19 transmissions, a reaction-diffusion system is employed to describe the epidemiological phenomenon through two processes: (i) A reaction process of SEIRS-type (Susceptible, Exposed, Infected, Recovered, and Susceptible) kinetics; and (ii) a diffusion process that models local dispersal of individuals through the incorporation of spatial dimension. Optimal control theory and numerical simulation studies are performed to examine the combined effects of reinfection, spatial dispersal process, and distinct control measures on the severity of the outbreaks. The results indicate the effectiveness of implementing adequate control measures vaccination and treatments to prevent further disease outbreaks and minimise the number of infected cases. The insights from these modelling studies benefit different stakeholders, e.g., public health practitioners and policymakers in devising sustainable COVID-19 management plans to prepare for endemicity and deal with large-scale disease outbreaks in the future.

Keywords: Sustainability, optimal control, COVID-19, infectious diseases, human disease.

Introduction

The intricate relationship between infectious diseases and human behaviours is a topic of significant scientific interest, as they constantly influence and shape each other. On one hand, the spread of diseases is heavily influenced by our actions, movements, interactions, and adherence to preventive measures. On the other hand, the emergence and progression of viruses can trigger alterations in our day-to-day behaviours as we adapt to the threat they pose. While this connection may seem intuitive, our understanding of these complex interactions remains limited in scope. Before the COVID-19 pandemic, studies predominantly focused on theoretical aspects, lacking substantial empirical evidence to validate their claims. However, the landscape drastically changed in 2020, marked by significant developments in this field (Perra, 2021).

Over the past few decades, numerous mathematical models have been proposed to assess the spread and control of infectious diseases. These models play a critical role in multiple fields, such as policy-making, risk evaluation, control strategy development, and improving various aspects of the health-economic relationship (Salman *et al*., 2021). The main purpose of these models is to employ compartmental relationships to characterise the infection status (susceptible and infected) and the transmission process between these states. These models are designed to divide the population into different compartments while incorporating certain epidemiological assumptions, e.g., the nature and duration of transitions between each compartment (Padmanabhan *et al*., 2021). The use of such models has demonstrated some key benefits in public health research focused on investigating various diseases and prospective prevention interventions (Libotte *et al*., 2020; Liu & Lou, 2022). For instance, some progress and developments have been made through research on measles vaccination (Bauch *et al*., 2009), HIV/AIDS (Mukandavire *et al*., 2009), tuberculosis (Bowong & Kurths, 2010), dengue (Weiss, 2013), and pertussis epidemiology (Pesco *et al*., 2014). Those models have significantly enhanced our understanding of the dynamics of infectious diseases, thus facilitating the development of efficient and sustainable prevention measures. The application of mathematical models has been instrumental in guiding the decision-making procedures associated with public health and healthcare preparedness strategies.

However, most of these models are formulated using Ordinary Differential Equation (ODE) systems, which are practical in modelling the transmission dynamics of infectious diseases. Their capacity to consider population movements and interactions is restricted. Integrating the spatial dispersal process into the modelling framework is of utmost importance. Partial Differential Equations (PDE) models present a robust method to capture the complexities of diseases in the actual world (Zhang *et al*., 2019; Wang *et al*., 2021). For instance, the principle of diffusion, when integrated into infectious disease models, enables us to account for the spatial propagation of infections across geographical regions (Zhu *et al*., 2021). This is particularly relevant for diseases involving significant degrees of contact and mobility among individuals, like COVID-19 (Ahmed *et al*., 2021; Liu *et al*., 2022). Consequently, the incorporation of the diffusion process using PDE models portrays the realism and reliability of disease modelling, which permits a more precise representation of real-world scenarios (Wang *et al*., 2012; Salman *et al*., 2023).

The present study aims to analyse a sustainable preventative measure to curb the outbreaks by employing pharmaceutical measures, such as administering vaccines and providing adequate treatment. The principal emphasis is achieving sustainable epidemic control (Zhu *et al*., 2021) while reducing transmission among local communities. In particular, we consider a situation wherein complete containment or eradication of the virus is unattainable owing to the reinfection factor and waning immunity (Salman *et al*., 2021). In this context, the optimal control strategy must minimise the objective function. On one hand, the number of disease-related deaths should be minimised by ensuring adequate treatment and healthcare capacities (Tang, 2020). On the other hand, long-term vaccination strategies must be implemented in the population to prevent a potential second epidemic outbreak through herd immunity (Abidemi *et al*., 2021; Khandker *et al*., 2021). To address this problem, we apply Pontryagin's Maximum Principle (Lenhart & Workman, 2007; Salman *et al*., 2022) to an extended reaction-diffusion SEIRS-type model specifically designed to capture relevant aspects of COVID-19. By utilising this approach, we aim to compute the optimal solution that optimises the control measures while considering the complex dynamics of the epidemic.

Methodology

In this paper, we examine the transmission dynamics of COVID-19 in Malaysia using a family of generalised SEIRS-type models (Abdelaziz *et al*., 2020; Annas *et al*., 2020; Khan *et al*., 2021; Kammegne *et al*., 2023). Our research incorporates the crucial factors of reinfection (Mohd *et al*., 2020) and local dispersal (Mohd, 2019; Mohd & Noorani, 2021), significantly contributing to more realism of disease outbreaks. In contrast to the typical systems of ODEs, which rely only on the time variable, PDEs consider the combined influences of both time and spatial domain. The consideration of spatial dimensions allows the PDE systems to capture the intricacies of the epidemiological and spatial phenomena under investigation. Our model framework is specifically tailored to study the sustainability of vaccination control strategies combined with treatment control. A plan is considered sustainable when it allows an entity to effectively control the virus over time without further tightening the initial internal confinement measures (Zhu *et al*., 2021). Additionally, our framework enables the optimisation of levels of the dispersal process, considering the reinfected cases and carrying capacity as conditional factors. By utilising optimal control theory, our study provides valuable insights into developing sustainable control measures for effectively combating COVID-19.

Model Formulation

The proposed model is represented by a system of nonlinear partial differential equations (PDEs), carefully designed to capture the intricate dynamics of the disease outbreak:

$$\begin{cases} S_t = d\,\Delta\,S + \mu N - \frac{\beta_1 SI}{N} + \beta_2 R - \frac{\mu NS}{K}, & x \in \Omega, \mathrm{t} > 0, \\ E_t = d\,\Delta\,E + \frac{\beta_1 SI}{N} - \varepsilon\mathrm{E} - \frac{\mu NE}{K}, & x \in \Omega, \mathrm{t} > 0, \\ I_t = d\,\Delta\,I + \varepsilon\mathrm{E} - (\gamma + \delta)\mathrm{I} - \frac{\mu NI}{K}, & x \in \Omega, \mathrm{t} > 0, \\ R_t = d\,\Delta\,R + \gamma\mathrm{I} - \beta_2\mathrm{R} - \frac{\mu NR}{K}, & x \in \Omega, \mathrm{t} > 0, \end{cases} \tag{1}$$

with boundary condition

$$[d\,\Delta\,S] \cdot \boldsymbol{n} = [d\,\Delta\,E] \cdot \boldsymbol{n} = [d\,\Delta\,I] \cdot \boldsymbol{n} = [d\,\Delta\,R] \cdot \boldsymbol{n} = 0,\ x \in \partial\Omega, \mathrm{t} > 0, \tag{2}$$

and initial conditions

$$S(x,0) = S_0 \ \geq 0, E(x,0) = E_0 \ \geq 0, I(x,0) = I_0 \ \geq 0, R(x,0) = R_0 \ \geq 0. \tag{3}$$

In model (2.1), the total population (N) are divided into four compartments: Susceptible (S), exposed (E), infected (I), and removal (R), each with their respective densities at different locations (x) and times (t). The diffusion coefficient (d) accounts for spatial movement, reflecting the dispersal of individuals within the population. The demographic (or birth) rate (μ) considers population growth, while the frequency of new infections within a given time, also known as the incidence rate, given by ($\frac{\beta_1 SI}{N}$), whereby (β_1) is referred to as the transmission coefficient. The transition (ε) rate represents the movement from the exposed to the infected compartment, while the recovery rate (γ) represents the average duration at which infected individuals recover from a disease over time. The death rate (δ) captures the mortality rate among infected individuals. The reinfection rate (β_2) accounts for the possibility of individuals being infected again after recovery.

Furthermore, our model incorporates carrying capacity (K) to account for population limits. The term ($\frac{\mu NS}{K}$) in Equation (2.1) signifies the adjustment of the susceptible compartment based on the population size relative to the carrying capacity. This ensures that the model reflects the realistic constraints on population growth and the spread of the virus within the available resources and space.

The spatial domain Ω is a bounded region with a smooth boundary ($\partial\Omega$), and we impose homogeneous Neumann boundary conditions (NBCs), as described in Equation (2.2). This choice of boundary conditions ensures that the migratory flow into or out of the spatial domain is restricted, making the model suitable for studying a closed population system. The initial conditions (Equation 2.3), where S_0, E_0, I_0, and R_0 represent the initial densities of the respective compartments.

Sustainable Control Measures

To effectively curb the transmission of the virus, it is imperative that the basic reproduction number ($\mathfrak{R}_0$) remains below unity. One common approach to achieve this is through implementing lockdowns and closing borders, but these unsustainable measures come at the cost of adversely affecting the country's economy and cannot be implemented for extended periods. Therefore, an essential alternative is to pursue immunity among communities. For this purpose, we denote the basic reproduction number as $\mathfrak{R}_0$, which can be calculated using the following formula:

$$\mathfrak{R}_0 = \frac{\beta_1}{\gamma + \delta},$$

It is well known that the observed reproduction number will always be 0 if the entire population is immune, regardless of $\Re_0$. When $\Re_0 < 1$, an entity has a confining policy for COVID-19 without the need for further vaccinating or relying on non-pharmaceutical interventions or lockdown.

Our methodology involves the application of optimal control theory to investigate these measures, with particular emphasis placed on two central control strategies: Vaccination and treatment. We have chosen the two strategies above because of their significant functions in regulating and mitigating disease epidemics. Treatment involves using antiviral drugs or additional healthcare services to reduce the intensity and duration of infections. This measure minimises the impact on individuals, mitigates potential complications, and prevents death. On the other hand, to protect against specific diseases, it is a common practice to enhance the immune system by implementing a vaccination strategy. By integrating vaccination and treatment strategies within our optimal control framework, we aim to examine their effectiveness in disease control. Moreover, the proposed model offers valuable insights into devising and executing sustainable control measures that balance the combined effects of movement and effective disease control through vaccination and treatment.

To achieve this, we develop Equation (2.1) by introducing two control variables: Vaccination and treatment (Salman *et al*., 2022). These variables depend on both spatial location (x) and time (t) within the spatial domain Ω. The vaccination variable is represented as $\xi(x, t)$, while the treatment variable is denoted as $\tau(x, t)$. As a result, the control model based on Equation (2.1) can be expressed as follows:

$$\begin{cases} S_t = d\,\Delta S + \mu N - \frac{\beta_1 SI}{N} - \xi S + \beta_2 R - \frac{\mu NS}{K}, & x \in \Omega, t > 0, \\ E_t = d\,\Delta E + \frac{\beta_1 SI}{N} - \varepsilon E - \frac{\mu NE}{K}, & x \in \Omega, t > 0, \\ I_t = d\,\Delta I + \varepsilon E - (\gamma + \delta + \tau) I - \frac{\mu NI}{K}, & x \in \Omega, t > 0, \\ R_t = d\,\Delta R + (\gamma + \tau) I - \beta_2 R + \xi S - \frac{\mu NR}{K}, & x \in \Omega, t > 0. \end{cases} \tag{4}$$

The boundary conditions are given by:

$$[d\,\Delta S] \cdot \boldsymbol{n} = [d\,\Delta E] \cdot \boldsymbol{n} = [d\,\Delta I] \cdot \boldsymbol{n} = [d\,\Delta R] \cdot \boldsymbol{n} = 0, \; x \in \partial\Omega, t > 0, \tag{5}$$

and initial condition defined on Hilbert space $\mathcal{H} = L^2(\Omega^2)$,

$$(IOC)\; S^0_{\tau,\xi},\, E^0_{\tau,\xi},\, I^0_{\tau,\xi},\, R^0_{\tau,\xi} > 0. \tag{6}$$

Now, we define the objective function:

$$J(\tau, \xi) = \int_0^T \int_\Omega C_1 I + C_2 \left[\frac{R}{K}\right]^m \xi + C_3 \tau^2 \; dxdt, \tag{7}$$

$$\mathcal{M} = \left\{J(\tau, \xi) \in L^1(0, T) \middle| \big(\tau(x,t), \xi(x,t)\big) \in \Omega \times [0, T]\right\}, \tag{8}$$

In Equation (2.7), C_1 represents the weightage assigned to the number of infected individuals while C_2 and C_3 represent the weights assigned to the control variables, respectively. The objective function reflects our aim to effectively control the outbreak by reducing the infection rate and increasing the recovery rate. With the presence of control variables $\tau(x, t)$ and $\xi(x, t)$ our objective is to find the optimal controls $\tau^*(x, t)$ and $\xi^*(x, t)$ that enable us to achieve this goal.

$$J(\tau^*, \xi^*) = \min\{J(\tau, \xi), \tau, \xi \in \mathcal{M}\} \tag{9}$$

Define the cost function:

$$R\big(0,\Theta(\mathrm{x},0)\big) = \min_{\tau(x,t),\xi(x,t)\in\mathcal{M}} J\big(0,\Theta(\mathrm{x},0)\,;\,\tau(x,t),\xi(x,t)\big) \tag{10}$$

Subsequently, we establish the adjoint system for the control Equation (3.1) by introducing the Hamiltonian function outlined below.

$$\begin{aligned} H(x,t,S,E,I,R,\tau,\xi,\lambda) &= C_1 I + C_2\left[\frac{R}{K}\right]^m \xi + C_3\tau^2 + \\ &\lambda_1\left[d\,\Delta S + \mu(S+E+I+R) - \beta_1\frac{SI}{(S+E+I+R)} - \xi S + \beta_2 R - \mu\frac{\big((S+E+I+R)S\big)}{K}\right] + \\ &\lambda_2\left[d\,\Delta E + \beta_1\frac{SI}{(S+E+I+R)} - \varepsilon E - \mu\frac{\big((S+E+I+R)E\big)}{K}\right] + \quad \textbf{(11)} \\ &\lambda_3\left[d\,\Delta I + \varepsilon E - (\delta+\gamma+\tau)I - \mu\frac{\big((S+E+I+R)I\big)}{K}\right] + \\ &\lambda_4\left[d\,\Delta R + (\gamma+\tau)I + \xi S - \beta_2 R - \mu\frac{\big((S+E+I+R)R\big)}{K}\right]. \end{aligned} \tag{11}$$

The following presents the adjoint system corresponding to the control Equation (2.4).

$$\begin{cases} \frac{d\lambda_1}{dt} = d\,\Delta S - \lambda_1\mu + (\lambda_1-\lambda_2)\beta_1 I\frac{N-S}{N^2} + (\lambda_1-\lambda_4)\xi \\ \qquad + \frac{\mu}{K}[(S+N)\lambda_1 + E\lambda_2 + I\lambda_2 + I\lambda_3 + R\lambda_4\,] \\ \frac{d\lambda_2}{dt} = d\Delta E - \lambda_1\mu + (\lambda_2-\lambda_1)\beta_1\frac{SI}{N^2} + (\lambda_2-\lambda_3)\varepsilon \\ \qquad + \frac{\mu}{K}[S\lambda_1 + (E+N)\lambda_2 + I\,\lambda_3 + R\lambda_4] \\ \frac{d\lambda_3}{dt} = d\Delta I - C_1 - \lambda_1\mu + (\lambda_1-\lambda_2)\beta_1 S\frac{N-I}{N^2} + \delta\lambda_3 + (\lambda_3-\lambda_4)(\gamma+\tau) \\ \qquad + \frac{\mu}{K}[S\lambda_1 + E\lambda_2 + (I+N)\lambda_3 + R\lambda_4] \\ \frac{d\lambda_4}{d??} = d\Delta R + nC_2\xi^2\frac{R^{m-1}}{K^m} - \lambda_1\mu + (\lambda_2-\lambda_1)\beta_1\frac{SI}{N^2} \\ \qquad + \frac{\mu}{K}[\lambda_1 S + \lambda_2 E + \lambda_3 I + \lambda_4(R+N)] - \beta_2(\lambda_1-\lambda_3), \end{cases} \tag{12}$$

Taking the partial derivatives of (2.11) with respect to τ and ξ, and substituting them into the optimal control solution λ′.

$$\frac{\partial H}{\partial \xi} = C_2\left[\frac{R}{K}\right]^m + (\lambda_4-\lambda_1)S.$$

$$\frac{\partial H}{\partial \tau} = 2C_3\tau + (\lambda_4-\lambda_3)I,$$

To achieve the minimum cost for the objective function, we set $\frac{\partial H}{\partial \tau} = \frac{\partial H}{\partial \xi} = 0$. By doing so, we obtain:

$$\begin{cases} \xi^* = \min\left(\max\left(0,\frac{S^*(\lambda_1-\lambda_4)}{2C_2\left[\frac{R^*}{K}\right]^m}\right),\xi_{max}\right), \\ \tau^* = \min\left(\max\left(0,\frac{I^*(\lambda_3-\lambda_4)}{2C_3}\right),\tau_{max}\right). \end{cases} \tag{13}$$

Results and Discussion

In this section, we conduct the numerical simulation to examine the effects of distinct preventive measures, e.g., vaccination and treatment, on spreading infectious diseases. A "forward-backwards sweep" iteration method can solve such optimality systems. Each sweep requires a PDE-solving technique, such as finite difference methods, with central differences commonly used for discretising spatial derivatives (Lenhart & Workman, 2007). The iterations are repeated until convergence is achieved, indicated by the proximity of successive control and state solutions. By considering the control variables in the model, we can evaluate the impact of different control measures on the spread and control of the epidemic. These simulations provide valuable insights into the effectiveness of various intervention measures and aid in decision-making processes. Through these numerical simulations, we aim to assess the performance of the SEIRS and the control models, providing valuable information for understanding the dynamics of COVID-19 and evaluating potential control strategies in the context of the specific situation in Malaysia.

Our results highlight the effectiveness of distinct pharmaceutical interventions aimed at ensuring that $\mathfrak{R}_0$ remains below unity, even in the presence of reinfection. Through our research, we have discovered that when these pharmaceutical intervention measures are sustainably implemented to control infectious disease spread and monitor the basic reproduction number ($\mathfrak{R}_0$), the transmission dynamics can be significantly curbed. Considering the estimated parameters based on the real COVID-19 cases in Malaysia, as shown in Table 1, we can calculate the value of the basic reproduction number, i.e., $\mathfrak{R}_0 = 1.6480$. Then, we will demonstrate that the proposed control measures can effectively reduce the $\mathfrak{R}_0$ below unity.

In Figure 1 (a), the focus is on the infected population, specifically examining the scenario where the assumption of reinfection force is relaxed ($\beta_2 = 0$). In comparison between the situation before (red curve) and after (blue curve), the implementation of these control strategies clearly shows a rapid reduction in the number of infected cases within several tens of days. This indicates that the control measures, including vaccination and treatment, have successfully limited the transmission of the disease. The decrease in infected cases illustrates the positive impact of these interventions in reducing the burden on healthcare systems and breaking the chains of infection. In order

Table 1: Parameter values for model (12)

Symbol	Description	Value
μ	Intrinsic growth rate	0.000006 (Salman *et al.,* 2021)
β_1	The transmission rate	0.18425 (Estimated)
β_2	The reinfection force	Vary (hypothetically)
ε	The rate of transition from exposed to infected	[0.55]
γ	The recovery rate	0.11178 (Estimated)
δ	The death rate	0.00002 (Salman *et al.*, 2021)
C_1	Weight for number infected	100,000-500,000 (Salman *et al.*, 2022)
C_2	Weight for vaccination	10,000-50,000 (Salman *et al.,* 2022)
C_3	Weight for treatment	10,000-50,000 (Salman *et al.*, 2022)
ξ	Max vaccination rate	0.5/day (Salman *et al.,* 2022)
τ	Max treatment rate	0.1/day (Salman *et al.,* 2022)
$S(0)$	Initial susceptible population	4,404,780 (Estimated)
$E(0)$	Initial exposed population	2,404,780 (Assumed)
$I(0)$	Initial infected population	293,652 (Estimated)
$R(0)$	Initial removal class population	3,174,938 (Estimated)

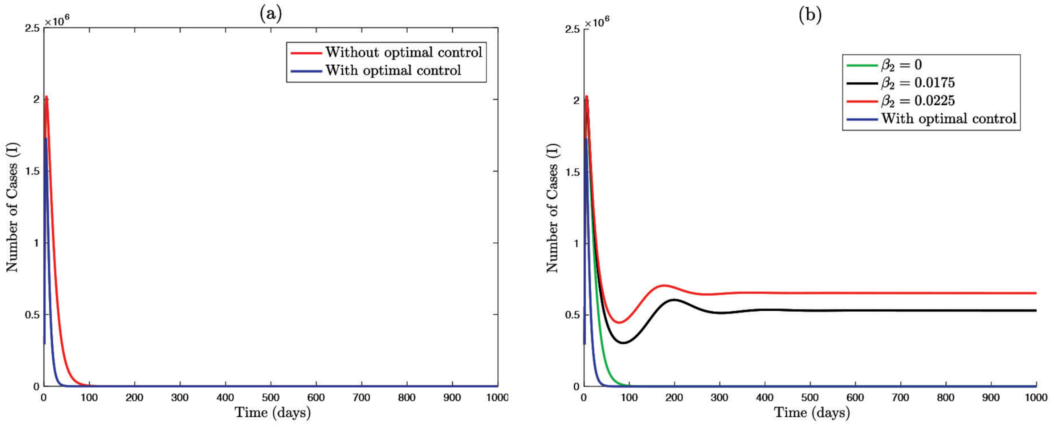

Figure 1: The diagram depicts the SEIRS-type epidemiological model exhibiting the dynamics of the infected population and the impact of incorporating pharmaceutical control measures, such as vaccination and treatment strategies, via an optimal control approach. The blue line signifies the scenario with control measures, while the red line connotes the situation without any control measures. Panel (a) portrays the dynamics with the assumption of reinfection is relaxed ($\beta_2 = 0$). In Panel (b), we analyse the impact of varying reinfection forces ($\beta_2 > 0$) by considering multiple cases. Table 1 provides the parameters employed for generating the plot. The figure provides a visual representation of the influence of control measures on the dynamics of the infected population and the efficacy of the selected strategies

to further evaluate the effectiveness of the control measures, we vary the reinfection force ($\beta_2 > 0$), Figure 1 (b) (black and respectively red curves) while maintaining the control strategy. We observe that the basic reproduction number becomes $\mathfrak{R}_0 = 0.8699$, signifying that the control measures used are robust and sustainable. This remarkable observation implies that the implemented control measures were sturdy and sustainable, as they could effectively restrain the disease to a manageable degree. This resilience is of particular significance due to the potential emergence of novel variants or the relaxation of preventive measures in real-world circumstances.

In the present study, an analysis was conducted to examine the effectiveness of control measures in curbing the transmission of COVID-19 among the exposed population. The findings from Figure 2 (a-b) indicate that the control measures successfully prevent the transition of individuals exposed to the infected states. The sustained control of the exposed population provides further evidence of how the implemented control measures can help to contain the spread of the disease.

In Figure 3, we examined the effects of vaccination and treatment on the susceptible and recovered populations, respectively. Figure 3 (a) reveals that vaccination is fundamental in reducing the number of susceptible individuals. This decrease helps cultivate a greater proportion of immune individuals among the populations, thereby restricting the reservoir of potential carriers for the virus. The findings underscore the significance of vaccination initiatives as a pivotal instrument in controlling disease propagation and attaining the required immunity levels. Figure 3 (b) investigates the importance of an adequate treatment strategy. The findings demonstrate that a successful treatment strategy hastened the recuperation process in infected patients. By reducing symptom severity and illness duration, treatment interventions can mitigate the burden on healthcare systems and lower the chances of complications or death. These outcomes emphasise the crucial need for timely treatment plans in sustainably managing the severity of the disease.

Overall, our numerical simulation investigation outcomes provide valuable insights regarding the effectiveness of control

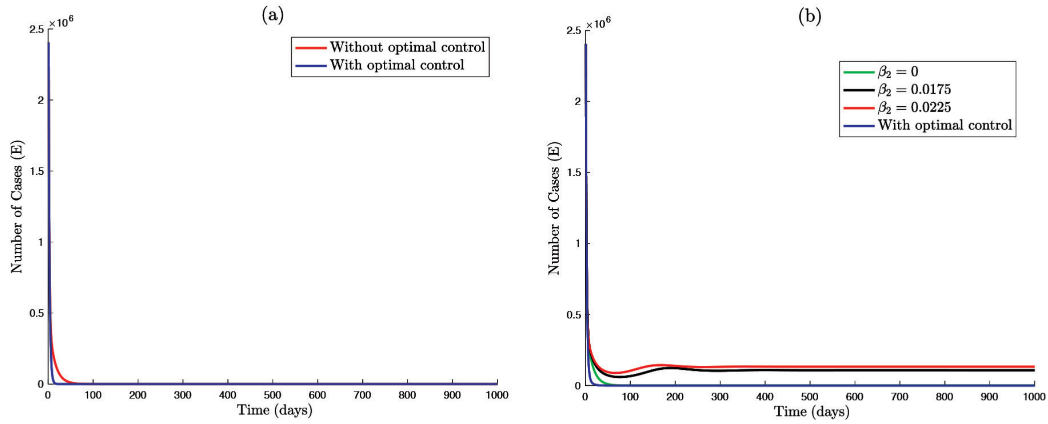

Figure 2: The SEIRS-type epidemiological model was utilised to create a plot demonstrating the dynamics of the exposed population. The incorporation of pharmaceutical control measures, specifically vaccination and treatment strategies, was showcased through an optimal control approach. The blue line represents the scenario with control measures, while the red line represents the scenario without any control measures. Panel (a) displays the dynamics with a relaxed reinfection parameter ($\beta_2 = 0$), while Panel (b) explores the impact of varying reinfection forces ($\beta_2 > 0$) through the consideration of multiple cases. The parameters used to generate the plot can be found in Table 1

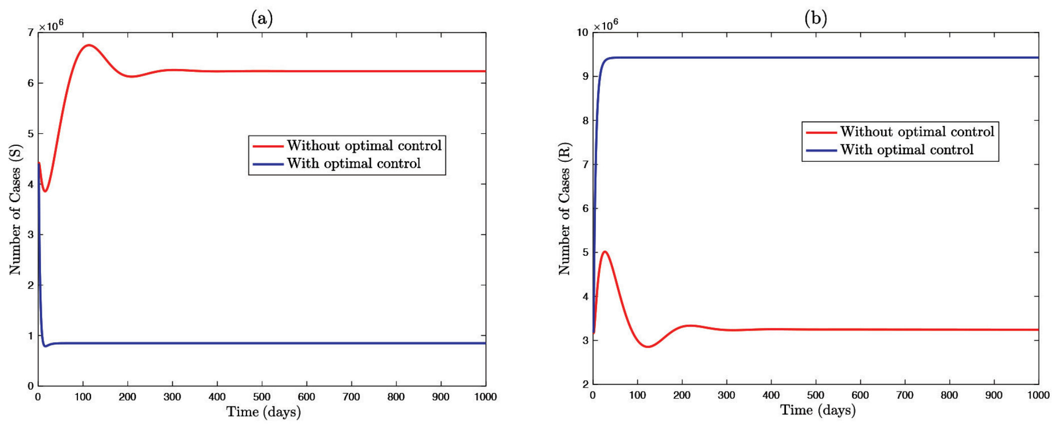

Figure 3: The plot demonstrates the dynamics of the susceptible (a) and recovered (b) populations using an SEIRS-type epidemiological model. It illustrates the impact of incorporating pharmaceutical control measures, such as vaccination and treatment strategies, through an optimal control approach. The blue line represents the scenario with control measures, while the red dashed line represents the situation without any control measures. The parameters utilised to generate the plot are provided in Table 1

measures, particularly vaccination and treatment, in curbing the transmission and repercussions of COVID-19 in Malaysia. These findings add to our understanding of evidence-based decision-making in controlling the spread of disease and can further guide public health policy planning and preparedness strategies to combat the recurrent COVID-19 epidemics. It is important to mention that this study simulation was conducted in a homogeneous environment, which may have some limitations. To enhance the robustness and generalisability of the findings, future investigations could delve into the effects of environmental heterogeneity. Such an examination will provide a better understanding of the crucial factors that influence the dynamics of the epidemiological system under consideration.

Conclusion and Epidemiological Implications

The computational simulations utilising the reaction-diffusion SEIRS model have yielded critical observations into the control of COVID-19 transmission dynamics in Malaysia. Our study reveals that implementing control measures has effectively mitigated the transmission of COVID-19 within both infected and exposed populations. The continuous control of the exposed group further corroborates the effectiveness of these interventions in containing the spread of the disease. Furthermore, a vaccination strategy has yielded promising results in reducing the number of susceptible individuals, thus facilitating the development of immunity within local communities and decreasing the potential rebound effects via several transmission waves. In addition, timely and efficient treatment interventions can accelerate the recovery process in the infected group, thereby alleviating the burden on healthcare resources and reducing the likelihood of adverse consequences or death. The capacity of the implemented control measures to remain effective despite reinfection forces underscores their sustainability and significance in mitigating the impact of the disease. This aspect is particularly crucial considering potential obstacles that may arise from newly emerging variants or due to the loosening of preventative measures in the future.

The results obtained from our numerical simulations possess significant epidemiological implications for the effective management of COVID-19, not only in Malaysia but also in analogous contexts globally. It is imperative to implement comprehensive control policies incorporating vaccination and treatment strategies to combat the transmission and consequences of this disease efficiently. The crucial role of vaccination initiatives in achieving immunity cannot be overstated, highlighting the importance of administering vaccines at the maximum feasible rate to reduce the number of susceptible individuals. This in turn enables healthcare resources to be allocated more efficiently and sustainably towards treatment while ensuring a significant proportion of the population is vaccinated and becomes immune. Our findings emphasise the necessity of vaccination in achieving immunity and its critical role in public health. A crucial aspect in mitigating the effects of a disease is to emphasise the importance of an efficacious and easily accessible treatment regimen. The implementation of treatment interventions can significantly decrease the severity and duration of symptoms, thereby mitigating the risk of overwhelming healthcare systems and reducing the likelihood of adverse outcomes.

In conclusion, our investigation provides significant perspectives on the effectiveness of control measures imposed to manage the spread of COVID-19, namely vaccination and treatment. These results can offer a foundation for decision-making based on robust evidence and direct public health endeavours towards containing the transmission of the disease and safeguarding communities from its repercussions.

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Conflict of Interest Statement

The authors declare that they have no conflict of interest.

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