

A NEW FUZZY MULTI-CRITERIA DECISION MAKING APPROACH FOR MUNICIPAL SOLID WASTE DISPOSAL OPTIONS

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Abstract: The decision on how to dispose municipal solid waste (MSW) is debatable, since there are many uncertain variables. Currently, this issue is discussed deeply among researchers and practitioners worldwide. This paper focuses on the development of the new multi-criteria decision-making (MCDM) approach with emphasizes on the various types of the input data in the evaluation process. By utilising both the analytic hierarchy process and fuzzy ideal solution method, the use of concentration and dilation concept was fully employed in the proposed method. The fundamental theory is briefly discussed to formulate the existing problem inline with the present dilemma of solid waste disposal. Validation through a case study was employed to demonstrate the applicability of the proposed method. The results showed that the model offers a versatile judgement, is clear in procedures and is successful in dealing with situations of diversified and too ill-defined in terms of input data. Moreover, it deals efficiently with the uncertainty of the initial information and facilitates systematic decision making.

KEYWORDS: Analytic hierarchy process (AHP), fuzzy ideal solution, linguistic hedges, multi-criteria decision making (MCDM) model, municipal solid waste (MSW)

Introduction

The rapid population growth, urbanisation process, economic growth and the rise in living standard generate high municipal solid waste (MSW) across the municipalities. With the current population of 27 million, Malaysia faces problems in MSW management and disposal. Despite concerted efforts in promoting recycling, reduction and reuse (3Rs) of materials, landfills remain an integral part of most solid waste management plans (Al-Jarrah and Abu-Qdais, 2006). Due to the increasing amount of MSW generated, more landfill is needed and the Malaysian government plans to open another two sanitary landfills at Seelong in Johor and Bukit Tagar in Selangor (Government of Malaysia, 2006) under the Ninth Malaysia Plan.

The most common practice of MSW disposal in Malaysia is open dumping (Lokman and Othman, 1992). This practice lacks precautionary environmental or health measures and is the easiest and cheapest method for removing waste. The Environmental Protection Society Malaysia (EPSM), reported that about eight to nine out of ten landfills practiced open burning in waste disposal (EPSM, 1979). The open dumps or dumpsites lead to degradation of the environment since they are susceptible to open burning, groundwater pollution and are exposed to scavengers and disease vectors (Kurian et al., 2005). In 2004 a total of 208 open burning cases were detected in disposal sites and significantly increased to 367 cases in 2005. On average 8.25 percent open burning cases occurred at disposal sites (DOE, 2004; 2005). In 1998, there were 230 waste disposal sites recorded and more than 78 percent or 181 sites practiced open dumping, while 49 sites were uncontrolled

landfilling. However, the volume of disposal sites decreased slightly to 161 in 2002 with only 77 sites or 47.8 per cent practiced open dumping and the remaining 84 sites were landfill with control tipping (MHLG, 2002). However, due to the increasing land cost, awareness of environment pollution, opposition from surrounding communities and other factors, the decision to upgrade the existing certain landfills is given attention in the Ninth Malaysia Plan.

Presently, the available landfill volume is limited and more than half of the existing landfills are approaching the maximum capacity. Open landfill has led to environmental pollution, leachate problems, odour disturbance, and opposition by the surrounding community. Poor site design, inadequate compaction, lack of leachate collection and treatment system and shortage of landfill covering are other problems experienced in developing countries. These problems arise since almost 90 per cent of all landfills are non-engineered open dumping disposal facilities (Ranaweera and Trankler, 2001). This situation will lead to health and environmental problems if the potential adverse impacts are not carefully monitored.

Several options are available, including using other disposal systems which include sanitary landfilling, incineration and composting. Since MSW management is becoming increasingly complicated, developing a precision model is vital. Several models have been developed in recent decades to support decision making in MSW management. International research has focused on the development of a sustainable MSW model (Hung et al., 2007; Klang et al., 2003; Daskalopoulos et al., 1998; Huhtala, 1997). Other researchers have developed an integrated interval-parameter fuzzy-stochastic mixed-integer linear programming (IFSMILP) model to apply in long-term planning of MSW management systems (Huang et al., 2001); multi-criteria analysis (MCA) to provide an accessible and transparent waste management decision making (Chung and Poon, 1996); decision-making model of MSW management to improve policy impact analysis to predict the possible impacts (Su et al., 2007), and Hedonic Price Method (HPM) (Eshet et al., 2007).

Many studies have also concentrated on developing decision support system in assessing the integrated waste management (IWM) applications or to arrive at sustainable goals. These include Life Cycle Inventory (LCI) tools (McDougall and Hruska, 2000); Municipal Solid Waste-Decision Support Tool (MSW-DST) (Thorneloe, et al., 2007); Integrated Risk Based Approach (IRBA) (Kurian et al., 2005); Value-Focused Thinking (VFT) (Chambal et al., 2003); Geography Information Systems (GIS) and Analytic Hierarchy Process (AHP) (Siddiqui et al., 1996); Elimination Et Choice by Translating Reality III (ELECTRE III) (Hokkanen and Salminen, 1997); and spatial decision support system (SDSS) (MacDonald, 1996). Other researchers have investigated the life cycle assessment (LCA) for MSW management from different point of views (Chaya and Gheewala, 2007; Villanueva and Wenzel, 2007; Bovea et al., 2007; Moberg et al., 2005). However, three models have often been used: MCDM and multi-objective programming (MOP), as well as LCA. MCDM is used for choosing the best among several alternatives by considering many criteria. For example, AHP method is employed for solving the environmental problem with multiple-criteria (Haastrup et al., 1998; Tran et al., 2002; Ho, 2008), TOPSIS method (Hwang and Yoon, 1981), and outranking method (Geldermann et al., 2000; Roy, 1991). MOP is used for choosing the locating sites and management strategies (Chang and Wang, 1996; Chang and Wei, 1999). Conversely, LCA is usually used to evaluate the environmental impact of the alternatives for MSW management (Powell et al., 1996; Barton et al., 1996; Finnveden, 1999; Powell, 2000; Eriksson et al., 2002).

Although MCDM is the commonly used model in solving MSW problems, current research rarely explores and utilises the MCDM model based on the linguistic hedge to assist the decision makers, specifically using the dilation and concentration concepts. It was believed that the proposed approach enables dealing with the variety of input data (i.e., crisp, quantitative, and linguistic) simultaneously and may improve certain shortcomings in the existing methods.

In this research, a MCDM model is developed. It encompasses AHP (Saaty, 1980) and Fuzzy Ideal Solution (FIS) (Chen, 2000) associated with an integrated concept towards the sustainability of MSW management in the Malaysian context. The remainder of this paper is organised as follows. The problem situation is examined in Section 2, followed by discussion on the background in Section 3. Section 4 focuses on the model development of the proposed model. Section 5 describes the MSW disposal decision which is used in a case study to illustrate the application of the proposed model. Lastly, the benefits of the proposed approach are discussed in Section 6.

The Problem Situation

The state of Selangor and the Kuala Lumpur Federal Territory (KLFT) recorded the highest MSW generation in Malaysia which was estimated at more than 2.32 million tonnes per year in 2005 MSW increases more than 3.5 per cent annually. This huge amount of waste cannot be absorbed totally by existing disposal sites which mostly practise traditional landfilling and open dumping. Thus, alternative disposal methods are needed and they include fully sanitary landfilling, incineration, composting and material recycling. Generally, the process to identify the most suitable disposal system involves various criteria. It also involves the multi-criteria consideration of multiple stakeholders. Hence, the federal government of Malaysia plans to implement another disposal system for long term solution. The development of a comprehensive model is essential with millions of households and businesses generating millions of tonnes of MSW yearly. However, the decision to choose the best alternative waste disposal system needs full participation and cooperation from various parties. Moreover, the decision process itself is uncertain and information as well as quantitative and qualitative factors are lacking. A comprehensive MCDM model should be developed, which can be used by the local authorities to make flexible and robust decisions for the long-term solution.

Fundamental Theory: Preliminaries

In this section, some basic definitions of fuzzy set are briefly reviewed based on Chen (2000), Zhang and Lu (2003), Sakawa (1993), Buckley (1985), Zimmermann (1991), Kaufmann and Gupta (1985), and Zadeh (1965;1975). The definitions and notations as given below will be used throughout the paper until otherwise stated.

Definition 1 A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ (Zadeh, 1965).

Definition 2 A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for any x_1, x_2 in X .

$$\mu_{\tilde{A}}[\lambda x_1 + (1 - \lambda)x_2] \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right), \quad -(1)$$

where $\lambda \in [0,1]$.

Definition 3 A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that $\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1$ (Chen, 2000).

Definition 4 A fuzzy number $\tilde{A}$ is a fuzzy subset in the universe of a real number R that is both convex and normal (Sakawa, 1993).

Definition 5 The α -cut of the fuzzy number $\tilde{A}$ is defined

$$\tilde{A}_\alpha = \{x_i : \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in R\}, \quad (2)$$

where $\alpha \in [0,1]$ and $\tilde{A}_\alpha$ is a non-empty bounded closed interval contained in X and it can be denoted by $\tilde{A}_\alpha = [a_\alpha^l, a_\alpha^u]$, and a_α^l and a_α^u are the lower and upper bounds of the closed interval, respectively (see Figure 1) (Zimmermann, 1991; Kaufmann and Gupta, 1985).

Definition 6 A triangular fuzzy number (TFN) $\tilde{A}$ as shown in Figure 2, can be defined by a triplet (a, b, c) . The membership function $\mu_{\tilde{A}}(x)$ is defined as (Kaufmann and Gupta, 1985):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x < b, \\ \frac{x-c}{b-c}, & b \leq x \leq c, \\ 0, & x > c \end{cases} \quad (3)$$

Definition 7 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. If $\tilde{a} = \tilde{b}$, then $a_1 = b_1, a_2 = b_2$ and $a_3 = b_3$ (Chen, 2000).

Definition 8 If $\tilde{a}$ is a triangular fuzzy number and $0 < a_\alpha^l \leq b_\alpha \leq a_\alpha^u \leq 1$ for any $\alpha \in [0,1]$, then $\tilde{a}$ is called a normalized positive triangular fuzzy number (Chen, 2000).

Definition 9 $\tilde{D}$ is called a fuzzy matrix, if at least any entry in $\tilde{D}$ is a fuzzy number (Buckley, 1985).

Definition 10 A linguistic variable is characterized by a quintuple $(x, T(x), U, G, \tilde{M})$ in which x is the name of the value; $T(x)$ denotes the term set of x , that is, the set of the name of linguistic value of x , with each value being a fuzzy variable denoted generically by x and ranging across a universe of discourse U which is associated with the base variable u ; G is a syntactic rule or generating the name X , of values of x . A particular X , that is name generated by G , is called a term; and $\tilde{M}$ is a semantic rule for associating with each X its meaning, $\tilde{M}(x)$ which is a fuzzy subset U (Zadeh, 1975).

Definition 11 A linguistic hedges or a modifier is an operation that modifies the meaning of a term more generally, of a fuzzy set. If $\tilde{A}$ is a fuzzy set then the modifier k generates the (composite) term $\tilde{B} = k(\tilde{A})$. The modifiers used frequently are (Cheng et al., 1999):

Concentration

$$\mu_{con(\tilde{A})}(x) = \left[\mu_{\tilde{A}}(x) \right]^w, \text{ where } w > 1 \quad (4)$$

Dilation

$$\mu_{dil(\tilde{A})}(x) = \left[\mu_{\tilde{A}}(x) \right]^{1/w}, \text{ where } w > 1 \quad (5)$$

Definition 12 The linguistic hedges and their approximate meanings are specifically classified as shown in Table 1.

Definition 13 Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ and be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between $\tilde{a}$ and $\tilde{b}$ as (Chen, 2000):

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \quad (6)$$

Definition 14 Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. The fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d(\tilde{A}, \tilde{B})$ approaches 0 (Chen, 2000).

It should be noted that, there are many other distance methods are proposed, but vertex method is an effective and simple method to calculate the distance between two TFNs (Chen, 2000; Zhang and Lu, 2003).

In this study the AHP (Saaty, 1980) and FIS (Chen, 2000) were used and both methods are widely accepted in multi-criteria decision making problem. AHP offers systematic approaches to alternative selection and justification problem by using the hierarchical structure analysis, while the FIS is the most preferred alternative based on the shortest distance from the positive idea solution and the longest distance from the negative ideal solution. Interested readers may refer to Ho (2008) for recent development of various AHP applications.

Methodology: Model Development and Formulation

Since the nature of the MSW disposal selection problem involves multiple factors and various consideration from different stakeholders point of views (e.g., government, experts, NGOs, etc.), the model must comply with the basic of integrated MCDM model requirements towards a sustainable MSW management. This section discusses the new approach of the MCDM model with the utilised linguistic hedges (i.e., dilation and concentration concept) in judgement process.

Establish the hierarchy structure

In order to formulate the problems much easier, we transform the actual problems into hierarchy structure (Figure 3). As shown in Figure 3, the structure have four levels of hierarchy consisting of objective or goal of the study at the first level, followed by some related criterion in the second level, which support to achieve the objective of the problems. The third level encompasses the sub-criteria for each criterion from level two, and the alternative lies in the last level (fourth level).

Calculate the performance score of the criteria and sub-criteria

Basically, the weight of each criterion and sub-criteria can be obtained by either directly assigning or indirectly using pair-wise comparisons. In this study the directly assign by linguistic variables and linguistic hedges was used (Table 1 and 3), for stakeholders to evaluate the importance of the criteria and sub-criteria. The main reason in using this direct assign are; i) convenient from multi-field stakeholders' background, ii) simple and user friendly and iii) effective approach to deal with the uncertainty of information. To deal with the quantitative and qualitative original data, two methods are utilised to derive the performance score (Cheng et al., 1999).

- Quantitative data - Using the expert's opinions to construct the membership function of the sub-criteria, and use the membership function to compute value scores (see Table 2).
- Qualitative data – Utilise fuzzy linguistic variables to derive its corresponding value to the TFNs (Table 3 and Figure 4).

Obtain the aggregated fuzzy weights of criterion

Assuming that a stakeholders group has S persons, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = \frac{1}{S} \left[\tilde{x}_{ij}^1 (+) \tilde{x}_{ij}^2 (+) \dots (+) \tilde{x}_{ij}^S \right] \quad (7)$$

$$\tilde{W}_j = (\mu_{dil/conA}(x))^w \quad (8)$$

where

$\tilde{x}_{ij}^s, \tilde{w}_j^s$ are the rating and weight of the S^{th} stakeholders, respectively, and $w = (\alpha^1, \alpha^2, \dots, \alpha^k)$ is a power of dilation or concentration of k criteria (Definition 12).

Construct the fuzzy decision matrix

Generally, the fuzzy multi-criteria decision matrix can be expressed as follows:

$$\tilde{D} = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{matrix}, \quad (9)$$

where $A_1, A_2, \dots, A_m$ are possible alternatives, $C_1, C_2, \dots, C_n$ are criteria with which performance of alternatives are measured using linguistic variables. These linguistic variables can be described by triangular fuzzy number, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$.

Normalisation

To avoid the complexity, the linear scale transformation is used to deal with criteria on different scales into a comparable scale. Thus, the TFN is normalized, resulting in, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$), in the decision matrix as the performance matrix denoted by $\tilde{R}$.

$$\tilde{R} = \left(\tilde{r}_{ij} \right)_{m \times n} \quad (10)$$

where

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{M}, \frac{b_{ij}}{M}, \frac{c_{ij}}{M} \right); i = 1, 2, \dots, n; \text{ and } M = \max_j c_{ij}, \quad j \in B \quad (11)$$

$$\tilde{r}_{ij} = \left(\frac{N - c_{ij}}{N}, \frac{N - b_{ij}}{N}, \frac{N - a_{ij}}{N} \right); i = 1, 2, \dots, n; \text{ and } N = \max_i c_{ij}, \quad j \in C \quad (12)$$

B is a set of benefit criteria, where the higher the value of $\tilde{r}_{ij}$ the better it is for DM, and C is a set of cost criteria, where the lower the value of $\tilde{r}_{ij}$ the better it is for the DM., and this TFN normalization method is to preserve the property in ranges of [0,1].

Determine the FPIS and FNIS

Consider the different importance of each criterion and different relative weights of the group decision makers, the weighted performance matrix was constructed by power of the weight vector by the decision matrix as:

$$\tilde{V} = \left[\tilde{v}_{ij} \right]_{m \times n}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n \quad (13)$$

where

$$\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)^{\tilde{w}_j}$$

Then, we define the fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution (FNIS, A^-).

$$A^* = \{v_1^*, \dots, v_m^*\} = \{(\max_j c_{ij} / j \in B), (\min_j a_{ij} / j \in C)\}, \text{ and} \quad (14)$$

$$A^- = \{v_1^-, \dots, v_m^-\} = \{(\min_j a_{ij} / j \in B), (\max_j c_{ij} / j \in C)\} \quad (15)$$

where, $\tilde{v}_j^* = (1, 1, 1)$ and $\tilde{v}_j^- = (0, 0, 0)$, $j = 1, 2, \dots, n$.

By definition 13 the vertex method was used to calculate the alternatives' performance index with reference to ideal solution (Hwang and Yoon, 1981). The shortest distance from the positive idea solution and the longest distance from the negative ideal solution is the most preferred alternative. Thus, the distance between each alternative and the FPIS and FNIS is calculated as:

$$d_1^* = \sum_{j=1}^n d \left(\tilde{v}_{ij}, \tilde{v}_j^* \right) \quad (16)$$

$$d_1^- = \sum_{j=1}^n d \left(\tilde{v}_{ij}, \tilde{v}_j^- \right) \quad (17)$$

where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$, and

$d(\cdot, \cdot)$ is the distance measurement between two fuzzy numbers.

Calculate the performance index

A performance index (PI) is defined to determine the ranking order of all alternatives once the d_i^* and d_i^- of each alternative A_i ($i = 1, 2, \dots, m$) have been calculated. Hence, the PI for each alternative as (Fenton and Wang, 2006):

$$PI_i = \frac{1}{2n} [d_i^- + n - d_i^*] \quad (18)$$

where $i = 1, 2, \dots, m$ and n is the number of criteria. The closer the PI_i obtained to 1 the better the alternative's performance.

Determine the ranking order

Based on the closeness performance index obtained from sub-section 4.7 the ranking order of the n alternatives, $A_1, A_2, \dots, A_n$ can be determined by descending order.

An Application of the Proposed Model: A Case Study

Background

The states of Selangor and KLFT have experienced rapid economic growth during the last two decades and have attracted the migration of rural population as well as foreign investment. The state of Selangor is divided into twelve local authorities consisting of two city councils, six municipality councils and four district councils, while KLFT is the capital city of Malaysia, administrated totally under a single local authority called Kuala Lumpur City Hall (KLCH). The detail of KLFT's demography is shown in Table 4. As a result, a disposal method such as incineration, composting and material recycling is the best alternative compromise in MSW management. This is because, the most common MSW disposal method in these areas is open dumping and landfilling in most districts except one fully-sanitary landfill existing in Bukit Tagar, Selangor.

Data Collection and Evaluation Method

A relay one month survey was made in April 2008. During the site visits, interviews were conducted with the top waste-management persons, site manager/engineers as well as relevant non-government organisations (NGOs). The objective of this interview was to get feedback on qualitative and quantitative aspects. However, local data are lacking. Thus, data from overseas are used as substitutes after consulting and discussing with the experts. The interview also gathered information on the following aspects i) the estimated costs of the different disposal system, ii) the lifetime of the system, iii) the environmental issues such as pollution, leachte, gas emission, etc., iv) the relevant legislation, v) the type of existing disposal sites, etc. The face-to-face interviews were conducted and recorded based on the questionnaires designed according to the AHP to obtain the preferences for various criteria, sub-criteria, and different disposal options from three stakeholders groups. The experts were also asked to estimate the impacts of each disposal option such as the technological aspects, environmental issues as well as economical perspectives.

Generally, the data of this research was gathered from these studied areas (if available) and divided into three main sources as follows:

- Primary data – Data gathered from local sources (internal) including the government official reports (e.g., Department of Environmental (DOE) annual report, Ministry of Housing and Local Government, etc.), reputable local research, practitioners (business firms) with experience in MSW management for more than 10 years in Malaysia;
- Secondary data – Data gathered from the latest literatures. In these cases, initially experts were consulted and their opinion obtained before possible adoption in the proposed model.
- Direct resources – Data obtained directly from the experts regarding the data such as the estimated lifetime of the composting system etc.

To ensure validity and the reliability of the data during the collection processes, (i.e., personal interview and site visits), the proven procedures were adopted as suggested by Reige (2003) as follows:

- Internal validity – Diagrams and illustrations prepared in the data-compiling phase were validated during the collection of data/information.

- External validation – The definition of the scope and boundary of the study are optimised during the study design. For instance, the generalisation of MSW disposal systems in the local context (i.e., Malaysia) could be achieved.
- Reliability – Data were recorded in as concrete a manner as possible, by comparing and observing the current situations with recognised best practices and interventions proven for local context, and
- Construct validity – Interviews and observations were made during the site visits to clarify some aspects of the respective MSW disposal practices.

The criteria and sub-criteria

The common criterion used to achieve sustainable management of MSW management are environmental, economic, social and technological factors (Hung et al., 2007); environmental, economic, social aspects, and political agenda (Finnveden et al., 2007), environmental impacts, economically affordability and socially acceptance (Morrissey and Browne, 2004) and environmental, economic and social (Klang et al., 2003). Hokkanen and Salminen, 1997 stressed factors such as political, employment and resource recovery, while Chung and Poon (1996) emphasized, socio-political impacts, and environmental impacts. Based on the above literatures and discussion with stakeholders, four main key criteria were proposed to evaluate each disposal system, i.e., economic aspects, environmental aspects, social aspects, and technological aspects. Economic aspects involve system capital costs, operating costs, system benefits, market channels, and land value. Environmental aspects include health problems, pollution issues, impact of the natural resources, and waste elimination ability. The social aspects encompass social acceptability, social welfare, and ease of administration/monitoring. Finally, technological aspects comprise lifetime, applicability and maturity of the system.

Numerical example via a case study

A case study was conducted in the state of Selangor and the KLFT, Malaysia. These two states received significant attention from the federal government to plan the possibilities of using the MSW disposal alternatives. This is because, the amount of MSW generations of these two states has tremendously increased. The federal government plans to allocate a huge amount of budgeting for the new MSW disposal system. Four possible disposal systems with controlled landfills, $\{A_1, A_2, A_3, A_4\}$ which may be considered are as follows: A_1 is a fully sanitary landfill, A_2 is an incinerator-equipped waste-to-energy facility with controlled landfill, A_3 is a composting facility with controlled landfill, and A_4 is material recycling with controlled landfill. However, four main criteria should be considered; $\{C_1, C_2, C_3, C_4\}$: C_1 is the economic factors, C_2 is the environmental impacts, C_3 is the social impact, and C_4 is the technology used by the systems. Three groups of stakeholders (i.e., each group at least comprises three persons) are involved and give their assessment based on their expertise and experiences. The finalised evaluation for each group represents the mutual consensus between them (Table 5). The stakeholders comprise of government agencies (s^1), experts (s^2), and non-government organisations (NGOs) (s^3). Based on the Section 4, the computational procedure is given as follows:

- Decision makers shall identify the prospective MSW disposal options, the criteria, sub-criteria and construct the hierarchical structure as shown in Figure 5.

- ii. Assign the input weights for selection criterion by linguistic expression based on the linguistic hedges (i.e., dilation and concentration concepts) in Table 1 and the results are shown in Table 5.
- iii. Assign the preference rating of alternatives versus various sub-criteria using direct assigned method as shown in Table 6.
- iv. Aggregate the weights of the criterion and fuzzy rating $\tilde{x}_{ij}$ to obtain the aggregated fuzzy weight w_j and alternative A_i under each criterion C_j , respectively.
- v. Unify the different units of both the qualitative and quantitative data and construct the normalised fuzzy decision matrix. Aggregate the weights of the criterion and fuzzy rating $\tilde{x}_{ij}$ to obtain the aggregated fuzzy weight w_j and alternative A_i under each criterion C_j , respectively. The fuzzy aggregated rating $\tilde{x}_{ij}$ with respect to alternative A_i under each criterion C_j is given in Table 7.
- vi. Multiply the weight-normalised fuzzy decision matrix to the relative importance weights for each of the stakeholders. Based on the stakeholders feedback (i.e., via questionnaire), the relative importance weighted for each stakeholders agreed as; $s^1 = 0.50$, $s^2 = 0.33$, and $s^3 = 0.17$, where $\sum_{i=1}^3 s^i = 1$. For example, the weighted normalised decision matrix corresponding to A_1 with respect to C_1 (x_{11}) can be calculated as

$$\begin{aligned} x_{11} &= 0.5 [(0.4304, 0.5478, 0.6922) + (0.3889, 0.5694, 0.7750) + (0.4333, 0.6333, 0.80) + (0.70, 0.7778, 0.80)] \\ &\quad + 0.33 [(0.3962, 0.5592, 0.7608) + (0.3357, 0.650, 0.8929) + (0.2857, 0.5238, 0.8095) + (0.6333, 0.7444, 0.80)] \\ &\quad + 0.17 [(0.3162, 0.5192, 0.9208) + (0.1071, 0.9286, 1.0) + (0.3704, 0.5185, 0.7037) + (0.2905, 0.4921, 0.7048)] \\ &= (0.40, 0.5469, 0.7531) \end{aligned}$$

Similarly, the full entry values of normalised weighted fuzzy performance matrix are given in Table 8.

- vii. Calculate the weighted normalised fuzzy decision matrix by utilising the dilation and concentration concepts. For example (from Table 5), the power of dilation/concentration for first criteria (C_1), $w_1 = (3/2 \times 3/2 \times 3/2)/3 = 3/2$ ($i = 1, 2, 3, 4$). Similarly, the linguistic hedge values of w_2 , w_3 , and w_4 can be obtained as $5/4$, $7/6$, and $2/3$, respectively. The results are given in Table 9.
- viii. Determine the FPIS and FNIS using Eq-(14) - (15), and calculate the distance of each alternative versus FPIS and FNIS, respectively. The results are shown in Table 10.
- ix. Calculate the performance index of each alternative using Eq-(18). For example, the performance index for the first alternative (A_1) is

$$PI_1 = \frac{1}{2n} [d_1^- + n - d_1^*] = \frac{1}{2(4)} [2.4220 + 4 - 1.8099] = 0.577$$

Similarly, the performance index for A_2 , A_3 , and A_4 can be obtained as 0.524, 0.513, and 0.550, respectively.

- x. Identify the ranking order of the alternative based on the maximum relative value of performance index.

According to the performance index from step (ix) above, the ranking order of the four systems is A_1 , followed by A_4 , A_2 , and the last ranking is A_3 . Obviously, the best option is system A_1 .

Conclusion

In determining the best disposal options MSW management, the MCDM model was observed as an efficient tool to deal with the complexity and uncertainty of information related to the MSW disposal decision. The dilation and concentration concept was utilised to give the criteria weights so that the decision makers (i.e., stakeholders) feel comfortable and the evaluation process is much easier. The model approach is very flexible and is able to deal with the various types of input data such as qualitative information, fuzzy data, and crisp data. Moreover, it offers the criteria judgement from the opposite perspectives using cost-benefit criteria to resolve the problem from the initial stages. In addition, the model allows the stakeholders to give judgement by using their intuition (i.e., linguistic expression) without being subject to the rigid measurement such as numerical values. Thus, the proposed approach as illustrated in this paper can generate the effective feasible solution as a decision support system, particularly in MSW disposal options. Although the proposed approach is specifically to resolve the MSW disposal options, it can be applied to other similar MCDM problems.

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Table 1: The hedge values of the specific dilation and concentration (Cox, 1994)

<i>Linguistic hedges</i>	<i>Meaning</i>	<i>Hedge type</i>	<i>Hedge values (w)</i>
<i>Very A</i>	Intensify a fuzzy region	Concentration	$[\mu_{very\ A}(x)]^{3/2}$
<i>Positively A</i>	Contrast intensification	Concentration	$[\mu_{positively\ A}(x)]^{5/4}$
<i>A</i> (i.e., no hedges)	-	-	$[\mu_A(x)]^1$
<i>Usually A</i>	Contrast diffusion	Dilation	$[\mu_{usually\ A}(x)]^{3/4}$
<i>Somewhat A</i>	Dilate a fuzzy region	Dilation	$[\mu_{somewhat\ A}(x)]^{1/4}$
<i>Between above linguistic hedges</i>	Intensify/contrast/dilate	Concentration/dilation	between above two hedges value range

Table 2: The crisp data of the operation cost (c_{12}) for four disposal system

Item (c_{12})	A_1	A_2	A_3	A_4	Membership function
Averaging of operation cost (in RM/tonnes)	150	125	187.5	737.5	$\mu_{OC} = \begin{cases} (900 - x)/1000, & 100 \leq x \leq 800 \\ 1, & x < 100 \end{cases}$

Notes: RM is Ringgit Malaysia; US\$1 = RM 3.25 (approx.)

Table 3: Linguistic variables for the ratings

Linguistic variables	Symbol	TFN
Very poor	<i>VP</i>	(0,0,1)
Poor	<i>P</i>	(0,1,3)
Medium poor	<i>MP</i>	(1,3,5)
Fair	<i>F</i>	(3,5,7)
Medium good	<i>MG</i>	(5,7,9)
Good	<i>G</i>	(7,9,10)
Very good	<i>VG</i>	(9,10,10)

Table 4: Demography of KLFT and state of Selangor

	Kuala Lumpur Federal Territory	State of Selangor
MSW generation (average million tonnes/year)	1.082	1.240
Capita per d ay (kg)	1.26	1.57
Population(in mil.)	1.62 (2006) 1.70 (2010)	4.87 (2005) 5.31 (2010)
Area (sq km)	243.65	796,084
Eco. growth (%)	n.a	7.9 (2006)

Notes: RM is Ringgit Malaysia; US\$1 = RM3.25(Approx.), n.a. not available

Sources: Government of M'sia(2006); State of Selangor Annual Report (2006)

Table 5: The important hedges of each of the criteria

Criteria	s^1	Stakeholders s^2	s^3
C_1	<i>Very important</i>	<i>Very important</i>	<i>Very important</i>
C_2	<i>Important</i>	<i>Positively important</i>	<i>Very important</i>
C_3	<i>Positively important</i>	<i>Important</i>	<i>Positively important</i>
C_4	<i>Somewhat important</i>	<i>Very important</i>	<i>Somewhat important</i>

Table 6: The importance weights of all criteria and sub-criteria from three stakeholder's perspectives

Item	s^1	A_1 s^2	s^3	s^1	A_2 s^2	s^3	s^1	A_3 s^2	s^3	s^1	A_4 s^2	s^3
<i>Economic (C_1)</i>												
c_{11} Capital cost (<i>in RM billion</i>)	(0.95, 1.0, 1.10)			(6.0, 6.25, 6.50)			(1.50, 1.95, 2.5)			(1.0, 1.25, 1.30)		
c_{12} Operation cost (<i>in RM/tonne</i>)	150			125			187.5			737.5		
c_{13} System benefit	MP	MP	P	G	G	VP	F	G	MG	F	G	G
c_{14} Market channel	F	MP	P	G	G	VP	MP	G	F	MP	G	F
c_{15} Land value	VP	VP	P	VP	F	MP	MP	MG	VP	MG	F	G
<i>Environmental (C_2)</i>												
c_{21} Human health	MP	I'	VP	I'	I'	VP	MP	I'	P	VG	MG	G
c_{22} Pollution issue	MP	P	VP	F	F	VP	MG	F	F	G	F	F
c_{23} Nature resource	G	P	VP	VG	G	VP	I'	I'	I'	VG	MG	MG
c_{24} Waste elimination	MG	MP	F	G	VG	G	MG	F	MG	F	MP	MP
<i>Social (C_3)</i>												
c_{31} Social acceptability	G	F	MG	F	F	VP	MP	F	G	G	VG	F
c_{32} Social welfare	MG	F	MG	MP	F	VP	P	F	G	F	VG	F
c_{33} Ease to admin/ monitoring	MP	P	VP	P	MP	VP	F	MG	G	G	F	G
<i>Technological (C_4)</i>												
c_{41} Lifetime (<i>in year</i>)	(9, 10, 12)			(18, 20, 25)			(8, 10, 12)			(20, 25, 30)		
c_{42} Adaptability	VG	G	F	G	G	VG	F	P	G	F	G	F
c_{43} Maturity	VG	VG	MP	MG	G	VP	MP	P	P	MP	G	F

Notes: RM is Ringgit Malaysia, US\$1 = RM3.25(approx.)

Table 7: The aggregated fuzzy decision matrix

		s^1	s^2	s^3
C_1	A_1	(0.4304,0.5478,0.6922)	(0.3962,0.5592,0.7608)	(0.3162,0.5192,0.9208)
	A_2	(0.4350,0.5227,0.5904)	(0.4950,0.6227,0.7104)	(0.1950,0.2827,0.4504)
	A_3	(0.4084,0.5968,0.7821)	(0.6456,0.7825,0.8763)	(0.4434,0.5492,0.6741)
	A_4	(0.3925,0.5274,0.6684)	(0.5325,0.6540,0.7417)	(0.5325,0.6540,0.7417)
C_2	A_1	(0.3889,0.5694,0.7750)	(0.3357,0.6500,0.8929)	(0.1071,0.9286,1.0)
	A_2	(0.3250,0.4750,0.6250)	(0.3750,0.5250,0.6750)	(0.1750,0.9750,1.0)
	A_3	(0.3056,0.5278,0.7500)	(0.1071,0.3929,0.6786)	(0.2817,0.5516,0.7857)
	A_4	(0.1071,0.2036,0.3750)	(0.1056,0.3167,0.6389)	(0.1500,0.3750,0.6250)
C_3	A_1	(0.4333,0.6333,0.80)	(0.2857,0.5238,0.8095)	(0.3704,0.5185,0.7037)
	A_2	(0.1905,0.4286,0.7143)	(0.3333,0.6190,0.9048)	(0.0,0.0,1.0)
	A_3	(0.1905,0.4286,0.7143)	(0.4074,0.6296,0.8519)	(0.70,0.90,1.0)
	A_4	(0.5667,0.7667,0.90)	(0.70,0.8333,0.90)	(0.4333,0.6333,0.80)
C_4	A_1	(0.70,0.7778,0.80)	(0.6333,0.7447,0.80)	(0.2905,0.4921,0.7048)
	A_2	(0.60,0.7556,0.9111)	(0.6667,0.8222,0.9444)	(0.50,0.5556,0.6444)
	A_3	(0.2794,0.4921,0.7048)	(0.0889,0.3333,0.80)	(0.3222,0.4444,0.5667)
	A_4	(0.4127,0.6587,0.9048)	(0.6889,0.8778,1.0)	(0.5079,0.7540,1.0)
Stakeholders weight		0.50	0.33	0.17

Table 8: The fuzzy normalized performance matrix and fuzzy weights of four alternatives

	C_1	C_2	C_3	C_4
A_1	(0.40,0.5469,0.7531)	(0.3242,0.6562,0.8518)	(0.3736,0.5777,0.7871)	(0.6095,0.7190,0.7841)
A_2	(0.4150,0.5160,0.6070)	(0.3167,0.5750,0.7042)	(0.2063,0.4206,0.8254)	(0.6056,0.7444,0.8778)
A_3	(0.4933,0.6508,0.7955)	(0.2354,0.4868,0.7321)	(0.3477,0.5742,0.8078)	(0.2230,0.4312,0.7135)
A_4	(0.4625,0.5907,0.7051)	(0.1138,0.2698,0.5046)	(0.5889,0.7667,0.8833)	(0.5206,0.7476,0.9524)
linguistic hedges	3/2	5/4	7/6	2/3

Table 9: The fuzzy weighted normalized performance matrix

	C_1	C_2	C_3	C_4
A_1	(0.250,0.4044,0.6536)	(0.2446,0.5905,0.8183)	(0.3171,0.5272,0.7563)	(0.7189,0.8026,0.8503)
A_2	(0.2673,0.3707,0.4730)	(0.2375,0.5007,0.6451)	(0.1586,0.3641,0.7994)	(0.7158,0.8214,0.9168)
A_3	(0.3465,0.5250,0.7095)	(0.1640,0.4066,0.6772)	(0.2916,0.5234,0.7795)	(0.3678,0.5708,0.7985)
A_4	(0.3145,0.4540,0.5920)	(0.0661,0.1945,0.4253)	(0.5391,0.7335,0.8653)	(0.6472,0.8237,0.9680)

Table 10: The distance measurement

	A_1	A_2	A_3	A_4
d^*	1.8099	2.0194	2.0816	1.8892
d^-	2.4220	2.2082	2.1860	4.2887

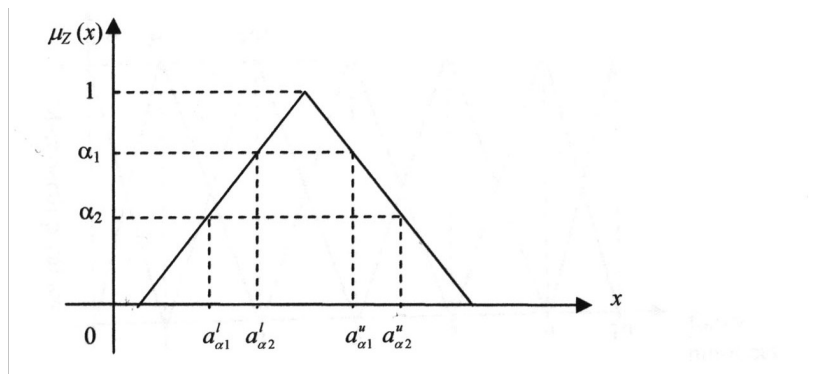
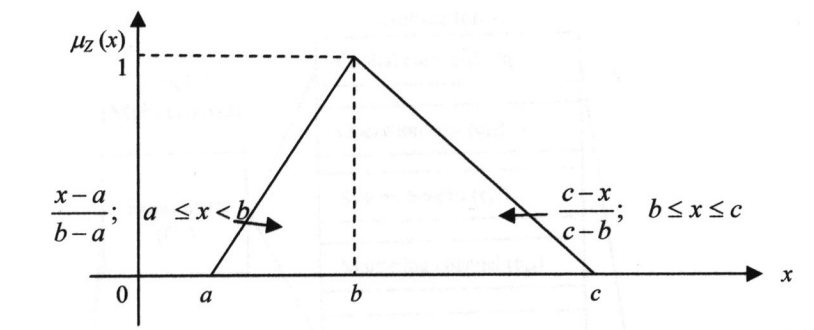
Figure 1: Fuzzy number $\tilde{A}$ with α -cutsFigure 2: A triangular fuzzy number $\tilde{A}$ 

Figure 3: The hierarchy structure

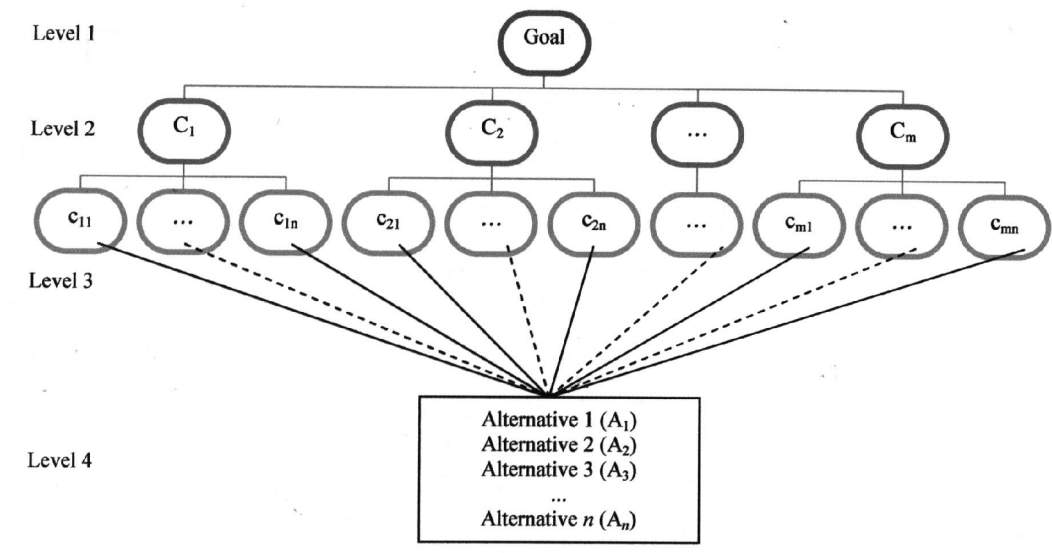


Figure 4: Membership functions of linguistic values for criteria ratings

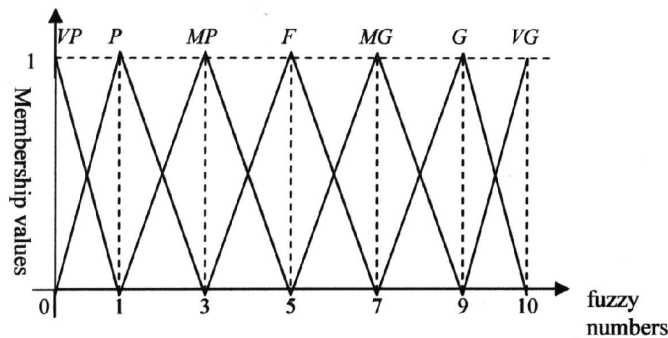
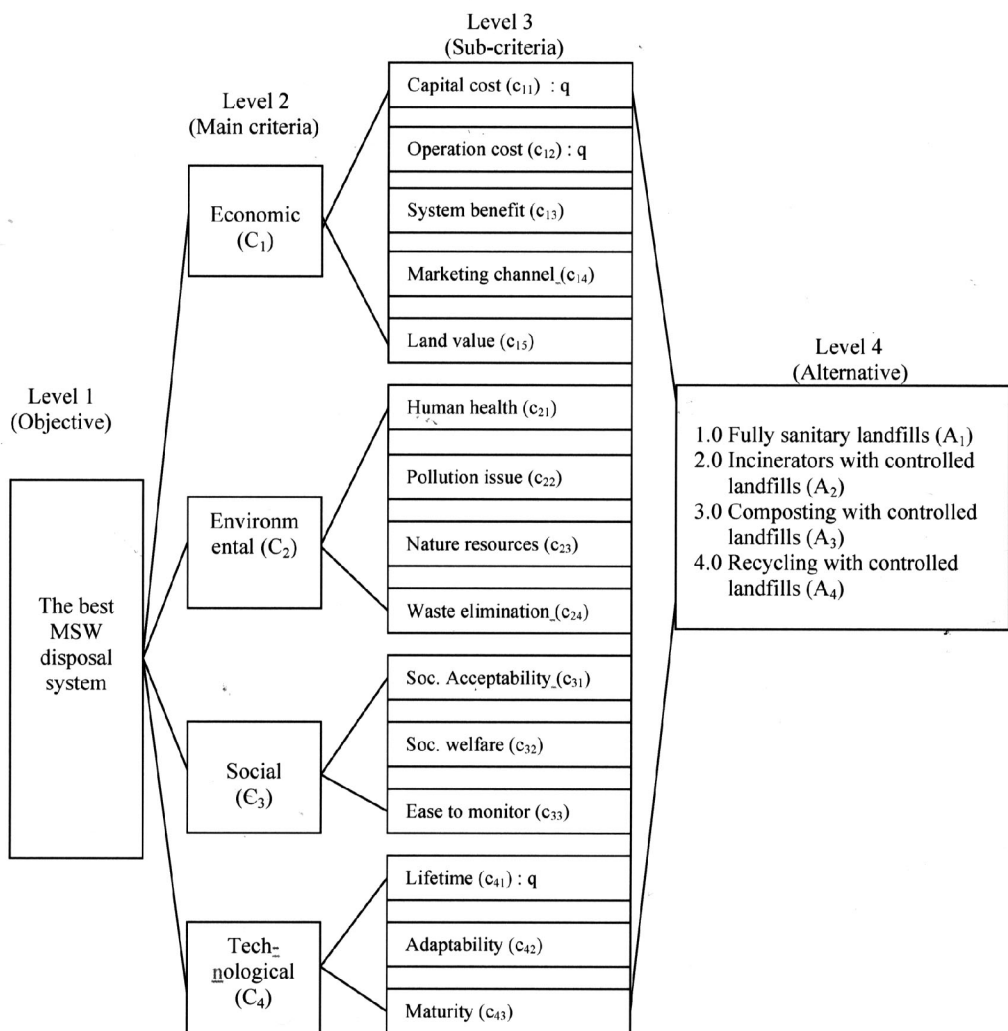


Figure 5: The hierarchical structure of MSW disposal alternatives



Note: q ~ quantitative data